Carleton UNIVERSITY
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Mathematics Tutorial Series

$$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

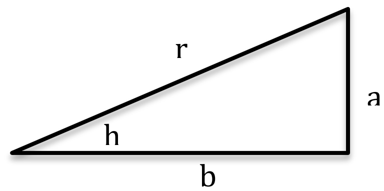
This limit is central to the use of trigonometric functions in calculus.

It is the reason that $\frac{d \sin x}{dx} = \cos x$.

It is why we can say that, when h is small, we may replace $\sin h$ with h.

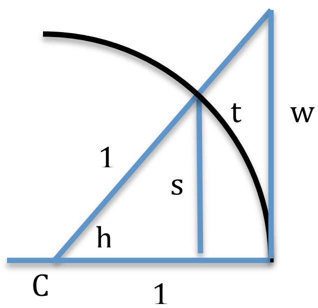
It requires that we measure angles in radians.

Recall the definitions of the trigonometric functions.
In a right triangle:

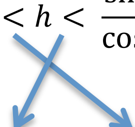


$\sin h = \frac{a}{r}$ $\cos h = \frac{b}{r}$ $\tan h = \frac{a}{b}$

Here
$s = \sin h$
$w = \tan h$
$t = h$



$$s < t < w$$
$$\sin h < h < \tan h$$
$$\sin h < h < \frac{\sin h}{\cos h}$$



$$\cos h < \frac{\sin h}{h} < 1$$

Now let $h \to 0$. Then $\cos h \to 1$ and we are done.

Therefore:

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

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